MORE FAMILIES OF FUNCTIONS [CN PAGE 75]

1. (a) $y = a\cos(bt^2)$ $\quad a > 0$ and $b > 0$

$y' = -a\sin(bt^2) \cdot 2bt$

$y'(1) = -2ab\sin b = 0$

$\sin b = 0$

$b = \pi$

$y'\left(\frac{1}{\sqrt{2}}\right) = -2ab\left(\frac{1}{\sqrt{2}}\right)\sin\left(b \cdot \frac{1}{2}\right) = 2$

$-a\pi\frac{2}{\sqrt{2}} \cdot \sin\left(\frac{1}{2}\pi\right) = 2$

$\left(\frac{-\sqrt{2}}{2\pi}\right)\left(-\frac{2}{\sqrt{2}}a\right)\pi = 2 \cdot \frac{-\sqrt{2}}{2\pi}$

$a = -\frac{\sqrt{2}}{\pi}$

$\boxed{y = -\frac{\sqrt{2}}{\pi}\cos\left(\pi t^2\right)}$

(b) $P(x) = ax^3 + bx^2 + cx + d$ $\quad a = 1$, C.P. at $x = 1$, POI at $(1, 4)$

$P'(x) = 3x^2 + 2bx + c = 0$ $\qquad P''(x) = 6x + 2b = 0$

$P'(2) = 12 + 4b + c = 0$ $\qquad P''(1) = 6 + 2b = 0$

$b = -3$

$12 + 4(-3) + c = 0$

$c = 0$

$P(1) = a + b + c + d = 4$

$1 - 3 + 0 + d = 4$

$d = 6$

$\boxed{P(x) = x^3 - 3x^2 + 6}$

(c) $a\left(1 - e^{-bx}\right) = Y$

$\lim\limits_{x \to \infty} a\left(1 - e^{-bx}\right) = \lim\limits_{x \to \infty} a\left(1 - \frac{1}{e^{bx}}\right) = a = 5$

$\therefore Y = 5\left(1 - e^{-bx}\right)$ WHERE $b \in \mathbb{R}$

2 $f(x)=(ax^2-b)^{1/2}$ a + b are constants >0

(a) DOMAIN:

$$ax^2-b \geq 0$$

$$x^2 \geq \frac{b}{a}$$

$$x \geq \frac{b}{a} \text{ OR } x \leq -\frac{b}{a}$$

$\boxed{\left(-\infty, -\frac{b}{a}\right] \cup \left[\frac{b}{a}, \infty\right) \text{ is the domain of } f(x)}$

(b)

$$f'(x) = \frac{1}{2}(ax^2-b)^{-1/2}(2ax)$$

$$f'(x) = \frac{ax}{(ax^2-b)^{1/2}} = \frac{ax}{\sqrt{ax^2-b}}$$

LOOK AT $\sqrt{ax^2-b}$; "a" and "b" are constants that become insignificant as $x \to \infty$.

$\therefore \sqrt{ax^2-b} \to x$ as $x \to \infty$

$\boxed{\text{Finally, } \lim_{x\to\infty} f'(x) = \lim_{x\to\infty} \frac{ax}{\sqrt{ax^2-b}} = 1}$

3 $r(x) = \left[a+(x-b)^2\right]^{-2}$

(a) The x-intercept occurs when $r(x)=0$.

But $r(x)$ can not be zero.

$\boxed{\text{Therefore, there is no } x\text{-intercept.}}$

(b) $r'(x) = -2\left(a+(x-b)^2\right)^{-3} \cdot 2(x-b) = 0$

$x = b$

$$r' \left\} \begin{array}{ccc} + & 0 & - \\ \hline \frac{1}{2}b & 0 & 2b \end{array} \right.$$

$$\left.\begin{aligned} r'\left(\tfrac{1}{2}b\right) &= -4\left(a+\left(\tfrac{1}{2}b-b\right)\right)^{-3}\left(\tfrac{1}{2}b-b\right) \\ &= -4(\ +\ )^{-3}\left(-\tfrac{1}{2}b\right) > 0 \\ r'(2) &= -4\left(a+(2b-b)^2\right)^{-3}(2b-b) < 0 \end{aligned}\right\} \boxed{x=b \text{ is a L. MAX}}$$